Plate 2.4^2 Extreme Point Rainfall of Varying Duration and Return Period

Introduction

Map 2.4^2 is a supplement to map 2.4 and contains maps of intense rainfall which differ from those of the original map only with respect to the interpolation method used. The reasons for adding map 2.4^2 are given below.

As can be seen from the text to map 2.4 [1], the so-called Kriging method, also called optimal interpolation, was used for interpolating the quantiles (rainfall values for a given return period). It was assumed that precise figures were available for the quantiles at the measuring stations, i.e. no data errors were allowed for in the interpolation model. In reality, the data values used are not exact measurements but estimated values for unknown true quantiles. It is possible, however, to take into account the uncertainty of the estimated values in the Kriging method [2,3]. Only then can the result be considered an optimal interpolation in the mathematical sense. In the case of the maps presented here the data values of the original map 2.4 have been used and re-interpolated, but taking into account the imprecision of the data.

Major differences between the two methods used

If data errors do not exist, or are not taken into consideration, the Kriging interpolation method reproduces the data values obtained by the measuring stations. This means that if an interpolation point is close to a measurement site then the interpolated value will be close to the measured value if the latter is taken into account in the interpolation. If, on the other hand, the error variation is taken into account in the Kriging method, the interpolated value will generally differ from the data value. In general: The larger the variance of the error at an observation point the smaller is the influence of this point on the value of an arbitrary interpolation point in the new map 2.4^2.

Results

The interpolation surface resulting from the extended method fits less closely the high values of stations which have short observation periods and which therefore are less exact. The new interpolation surface shows less variation and fewer marked local maxima that are unrealistic from a meteorological point of view (fig. 1). It should be pointed out that the results were not smoothed in any way. The interpolation was carried out point by point using the Kriging method as described below, without any further modification. In addition, it is impossible to obtain this result by taking into account a nugget effect, since this would have a uniform impact on all the stations. The nugget effect does not take into account the varying degree of precision among the stations.

Application

The instructions for the practical use of the maps as described for map 2.4 are also valid for map 2.4^2.

Types of error

The error at a measuring station is mainly the result of two factors: firstly a «real error», due to possible imprecisions (measurement error, station unrepresentativeness, processing error), and secondly the «random deviation» that results from the randomness of the weather patterns during the specific observation period of each station.

Random deviations are in particular larger and more obvious when the observation period is short or when neighbouring stations have periods that are a long time apart. The individual errors and deviations are unknown. It is possible, however, to estimate the variance and, under certain circumstances also the covariance, of the random deviation and to take them into account in the interpolation process.

Methodology

Using:

Z_i: quantile at the i-th station (true value), i = 1,..., N

$Z_i + e_i$: estimated value for Z_i replacing the unavailable measured value, estimated on the basis of the annual rainfall maxima for a series of years («reference period»)

e_i: deviation of the estimated value from the unknown true quantile value

Z_0: quantile at the interpolation point (true value)

the interpolator $\hat{Z}_0$ for Z_0 is as follows:

$$\hat{Z}_0 = \sum_{i=1}^{N} \lambda_i \left(Z_i + e_i \right)$$

If, in addition to the usual Kriging assumptions, one includes the obvious assumption: $\mathrm{Cov}(Z_i,e_k) = 0$ for all i, k (the variance of e_i may depend on Z_i) one obtains the following as a Kriging equation system for determining the interpolation factors λ_i:

$$\sum_{i=1}^{N} \lambda_i \left(\mathrm{Cov}\left(Z_i, Z_k\right) + \mathrm{Cov}\left(e_i, e_k\right) \right) - \mu = \mathrm{Cov}\left(Z_0, Z_k\right) \quad , k = 1, \ldots, N$$

$$\sum_{i=1}^{N} \lambda_i = 1$$

In the formula for the covariance (and variance) of data errors, a parameter α has been introduced that takes into account a real data error:

$\mathrm{Cov}(e_i,e_k) = (1 + \alpha\delta_{ik})\, \sigma_i\sigma_k\rho_{ik}$ with $\delta_{ik} = 1$ for $i = k$ and 0 for $i \neq k$

If the error consisted only of the weather-dependent variation, Variable $\alpha = 0$. σ_i signifies the root of the variance of this weather-dependent variation. It is dependent on the distribution of the annual rainfall maxima and the length of the reference period. The factor

$$\rho_{ik} = e^{-\beta h_{ik}} s_{ik} / v_{ik}$$

was introduced for the spatial correlation function h_{ik} signifies the distance, s_{ik} the length of the intersection interval and v_{ik} the length of the union interval of the reference periods of stations i and k. The parameter β denotes the decrease in the correlation to distance.

The parameters α and β were first estimated roughly using cross-validation, the (known) interpolation error $Z_0 + e_0 - \hat{Z}_0$ being standardised for the given station by dividing by its standard deviation τ_0 (the given station is indicated temporarily by 0 because it is not used as a support point for the interpolation). The following formula can therefore be used to obtain the square of the standard deviation:

$$\tau_0{}^2 = \mathrm{Var}\big(Z_0\big) + \mathrm{Var}\big(e_0\big) + \mu - \sum_{i=1}^{N} \lambda_i \Big(\mathrm{Cov}\big(Z_i, Z_0\big) + 2\,\mathrm{Cov}\big(e_i, e_0\big)\Big)$$

The standardisation means that the process is not susceptible to extreme values, although it does not provide a clear-cut result. This can be obtained, however, by adapting the parameters in an empirical variogram of the incorrect function.

If the expression 2 γ_z represents the variogram of the exact quantile Z the following formula can be used for the variogram of the incorrect quantile:

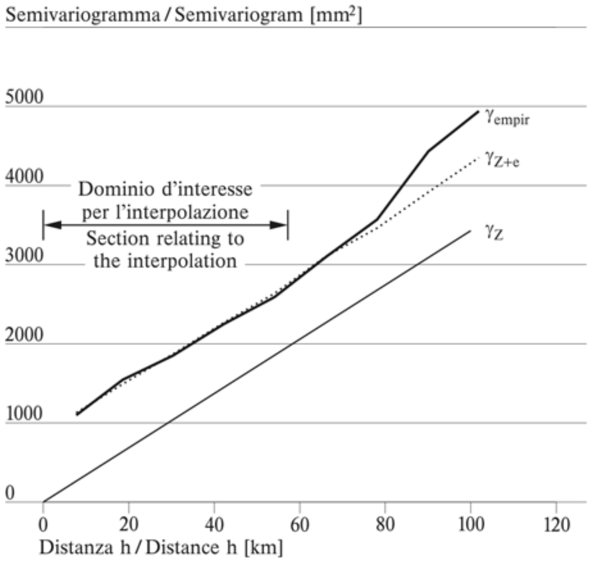
$$\mathrm{Var}\,(Z_i + e_i - Z_k - e_k) = 2\,\gamma_z + \mathrm{Var}\ e_i + \mathrm{Var}\ e_k - 2\,\mathrm{Cov}(e_i, e_k)$$

In the zone that is of interest here (0–60 km) the empirical variogram is linear, so that $\gamma_z\,(h) = c \bullet h$ can be inserted (fig. 2). This means that three unknown parameters must be estimated. Depending on the summation interval and the return period, the following values are obtained:

1 hour, 2.33 years:	c = 0.38,	α = 3.0,	β = 0.1
1 hour, 100 years:	c = 4.7,	α = 0.4,	β = 0.1
24 hours, 2.33 years:	c = 6.6,	α = 2.0,	β = 0.1
24 hours, 100 years:	c = 34.0,	α = 0.2,	β = 0.1

Fig. 2
Semivariogramma dei quantili – Durata di precipitazione: 24 ore, Tempo di ritorno: 100 anni

Semivariogram for the quantile – Rainfall period: 24 hours, return period: 100 years



γ_{empir} Incrementi quadratici medi all'interno delle varie classi distanziometriche

Mean values incremental squares within the various distance divisions

γ_{Z+e} Medie dei valori attesi degli incrementi quadratici all'interno delle varie classi distanziometriche

Mean expected values incremental squares within the various distance divisions

γ_{Z} Semivariogramma stimato del quantile effettivo non affetto da errori

Estimated semivariogram for the correct (true) quantile

I parametri del modello per la covarianza sono stati scelti in modo da ottenere una sostanziale coinzidenza di γ_{empir} e γ_{Z+e}. Si osservi comunque che per l'interpolazione non vengono impiegati i valori medi γ_{Z+e}, bensì valori singoli relativi a coppie di stazioni.

The parameters for the covariance model were chosen in such a way that γ_{empir} and γ_{Z+e} are as close as possible. It should be noted, however, that the individual values for each pair of stations were used for the interpolation, and not the mean values γ_{Z+e}.

References

[1] **Geiger, H. et al. (1992):** Extreme Punktregen unterschiedlicher Dauer und Wiederkehrperioden 1901–1970. In: Hydrologischer Atlas der Schweiz, Tafel 2.4. Bern.

[2] **Jensen, H. (1986):** Regionalisierung der Verteilungsfunktion des jährlichen Maximums des Tagesniederschlages im Kanton Zürich. Zürcher Geographische Schriften, Nr.27. Zürich.

[3] **Villeneuve, J.–P. et al. (1979):** Kriging in the Design of Streamflow Sampling Networks. In: Water Resources Research Vol.15, No. 6:1833–1840, Washington, D.C.